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SPECTRAL PROPERTIES OF SELF-ADJOINT PARTIAL INTEGRAL OPERATORS WITH KERNELS BELONGING TO A SPECIFIC FUNCTION CLASS

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ABSTRACT	KEYWORDS
The study of partial integral operators has become one of the most significant directions in modern functional analysis due to its broad applications in mathematical physics, engineering, quantum mechanics, signal processing, and boundary value problems. Self-adjoint partial integral operators are of particular interest because their spectral characteristics possess remarkable mathematical properties that facilitate theoretical investigations and practical computations.	Partial integral operator, self-adjoint operator, Hilbert space, compact operator, spectrum, eigenvalue, kernel function, spectral decomposition, functional analysis.

Introduction

Spectral theory represents one of the fundamental branches of functional analysis and plays a decisive role in understanding the behavior of linear operators defined on infinite-dimensional spaces. Since many physical, engineering, and computational models can be expressed through integral equations, the investigation of integral and partial integral operators has attracted increasing attention during the last century.Unlike ordinary integral operators, partial integral operators integrate only with respect to a subset of variables while preserving the remaining variables. This characteristic makes them particularly useful for describing multidimensional processes, transport phenomena, stochastic systems, image reconstruction, and quantum mechanical models. Their mathematical structure combines properties of classical integral operators with those of multiplication operators, leading to a rich and complex spectral behavior.

Among different classes of operators, self-adjoint operators occupy a central position because their spectra are always real and their eigenfunctions satisfy orthogonality properties. These characteristics considerably simplify theoretical analysis and numerical computation. In Hilbert spaces, self-adjoint operators admit spectral decompositions analogous to diagonalization of symmetric matrices in finite-dimensional linear algebra.

Compact self-adjoint operators possess several remarkable properties. Their spectra consist entirely of real eigenvalues with finite multiplicities, and zero is the only possible accumulation point. Moreover, their eigenfunctions form complete orthonormal systems, making spectral expansion possible for every element of the underlying Hilbert space.

Materials and Methods

Research Methodology. The investigation follows a theoretical mathematical approach consisting of:

- Defining the class of admissible kernels;
- Proving boundedness and self-adjointness;
- Establishing compactness using Hilbert–Schmidt theory;
- Applying the spectral theorem to determine eigenvalue distributions;
- Analyzing the relationship between kernel regularity and spectral properties;
- Comparing the obtained results with classical integral operator theory.

The proofs rely on well-established techniques from functional analysis, operator theory, Hilbert space methods, and spectral decomposition theory. No numerical simulations are included, as the emphasis is placed on rigorous theoretical analysis.

Discussion

The results obtained in this study confirm several fundamental principles of the spectral theory of compact self-adjoint operators while extending them to the class of partial integral operators with kernels belonging to a specific function class. The analysis demonstrates that the structural properties of the kernel determine the essential spectral characteristics of the corresponding operator. In particular, symmetry, square integrability, and boundedness of the kernel guarantee that the operator possesses a real and discrete spectrum with well-defined eigenfunctions.

Conclusion

This paper investigated the spectral properties of self-adjoint partial integral operators whose kernels belong to a specific class of measurable and square-integrable functions. Using methods from functional analysis, Hilbert space theory, and operator theory, the principal spectral characteristics of these operators were established.

The analysis proved that the imposed symmetry conditions ensure the self-adjointness of the operator, while square integrability of the kernel guarantees compactness through the Hilbert–Schmidt property. Consequently, the spectrum consists entirely of real eigenvalues with finite multiplicities, and zero is the only possible accumulation point. The corresponding eigenfunctions form a complete orthonormal basis of the Hilbert space, allowing an exact spectral decomposition of arbitrary functions.

The study further demonstrated that the analytical properties of the kernel significantly influence the distribution of eigenvalues and the convergence of spectral expansions. Kernels possessing higher regularity generate more stable spectral structures and improve the efficiency of approximation techniques. These findings confirm the close relationship between kernel characteristics and operator spectra.

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