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SPECTRAL PROPERTIES OF SELF-ADJOINT PARTIAL INTEGRAL OPERATORS WITH KERNELS BELONGING TO A SPECIFIC FUNCTION CLASS

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ABSTRACT

Spectral theory has become one of the fundamental branches of functional analysis due to its wide range of applications in mathematics, physics, engineering, and computational sciences. The concept of spectrum provides valuable information about the behavior of linear operators and offers effective analytical tools for solving differential and integral equations.Among various classes of linear operators, self-adjoint operators occupy a central position because of their remarkable mathematical properties. Their spectra consist entirely of real numbers, and the associated eigenfunctions form orthogonal systems in Hilbert spaces. These characteristics simplify both theoretical investigations and numerical computations.Partial integral operators represent a natural generalization of classical integral operators and frequently appear in multidimensional mathematical models. They arise in the study of heat transfer, wave propagation, elasticity theory, quantum mechanics, image reconstruction, and stochastic processes. Their mathematical analysis requires a combination of operator theory, measure theory, and functional analysis.

KEYWORDS

Introduction

The kernel function is the most important component of a partial integral operator. Depending on its continuity, symmetry, smoothness, and integrability, the operator may possess different spectral characteristics. When the kernel belongs to a suitable function class and satisfies symmetry conditions, the corresponding operator becomes self-adjoint, allowing the application of the classical spectral theorem.Recent developments in operator theory have focused on describing the spectrum of partial integral operators under different assumptions on kernel regularity. Researchers have investigated eigenvalue asymptotics, spectral approximation techniques, compactness criteria, and numerical algorithms for spectral computation. These studies have significantly contributed to solving complex mathematical models appearing in applied sciences.

The primary objective of this research is to analyze the spectral properties of self-adjoint partial integral operators with kernels belonging to a specific function class. Particular emphasis is placed on compactness conditions, spectral decomposition, eigenvalue distribution, and the theoretical foundations necessary for practical applications in modern mathematical analysis.This study also highlights the importance of spectral methods in solving Fredholm integral equations, boundary value problems, and various operator equations encountered in mathematical physics. The obtained results provide a theoretical basis for further investigations of multidimensional integral operators and their applications in computational mathematics.

Theoretical Background of Self-Adjoint Partial Integral Operators

The theory of partial integral operators is one of the most important areas of modern functional analysis. These operators establish a connection between classical integral equations and multidimensional operator equations, providing an effective mathematical framework for studying complex physical and engineering systems. Their spectral behavior is largely determined by the analytical properties of the kernel function and the underlying Hilbert space.

Which guarantees that all eigenvalues of the operator are real. This property distinguishes self-adjoint operators from general linear operators and makes them particularly suitable for spectral investigations.

Compactness is one of the most significant properties in spectral theory. If the kernel is continuous or belongs to the Hilbert–Schmidt class, the corresponding partial integral operator is compact. Compact self-adjoint operators possess a discrete spectrum consisting of real eigenvalues that accumulate only at zero. This result is one of the cornerstones of modern operator theory.

The regularity of the kernel has a direct influence on spectral properties. Smooth kernels generally produce rapidly decreasing eigenvalues, improving the convergence of spectral expansions. Conversely, kernels with lower regularity may lead to slower convergence and more complicated spectral structures. Consequently, the classification of kernels according to their functional properties has become an active research topic in operator theory.From an applied perspective, self-adjoint partial integral operators appear naturally in quantum mechanics, elasticity, heat conduction, acoustics, fluid dynamics, and electromagnetic theory. In these applications, eigenvalues often represent physical quantities such as energy levels or vibration frequencies, while eigenfunctions describe the corresponding physical states or modes.

These theoretical foundations provide the basis for a deeper investigation of spectral properties, eigenvalue distributions, and practical computational methods, which are discussed in the following sections of this study.

Spectral Properties of Self-Adjoint Partial Integral Operators

The spectral analysis of self-adjoint partial integral operators is a central topic in modern functional analysis because it provides a complete description of the operator through its eigenvalues and eigenfunctions. The spectrum determines the qualitative behavior of operator equations and forms the theoretical foundation for many applications in mathematical physics and numerical analysis.

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