AMERICAN JOURNALS
Open Access, Peer Reviewed Journals
American Journal of Technology and Applied Sciences
ISSN (E): 2832-1766 Volume 45, February - 2026

SOLUTE TRANSPORT IN A POROUS MEDIUM WITH PIECEWISE HOMOGENEOUS SALINIZATION

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ABSTRACT

In the paper a problem of salt transport in a porous medium with picewise salinization is numerically solved. Two cases are considered: 1) bed with immoile liquid zone saturated with salt solution; 2) dry bed with picewise salinization. Influence of salt adsorption on salt transport characteristics is established.

KEYWORDS

Adsorption of a substance, hydrodynamic dispersion, zones with mobile and immobile fluid, mass transport, porous medium.

Introduction

The study of solute transport problems in porous media is of great importance. During water flooding of oil reservoirs, the injected water may react with certain components of the reservoir rock, causing these components to dissolve and migrate into the aqueous phase. On the one hand, this alters the pore space structure and, consequently, the filtration properties of the reservoir; on the other hand, it changes the composition and properties of the injected water beyond the interaction zone with the rock. A similar problem arises in the underground leaching of rocks during the extraction of ore minerals [1].

The composition and structure of reservoirs are generally heterogeneous at both microscopic and macroscopic scales. Macroscopic heterogeneity in modeling fluid flow and solute transport processes is represented schematically in various ways. The most common representations include layered and zonally heterogeneous structures, particularly piecewise homogeneous ones. It should be noted that heterogeneity is understood both in terms of lithological composition and filtration–capacity properties of the reservoir.

In [2], solute transport in a porous medium with piecewise homogeneous salinization was considered. When water interacts with the rock, it dissolves salt components that enter the aqueous phase. As the water subsequently moves through the reservoir, the dissolved salts are transported together with the flow. The transport mechanism is generally convective–diffusive.

In the present paper, solute transport in a porous medium with piecewise homogeneous salinization of the rock is studied, taking into account adsorption of the dissolved substance on the rock surface. The adsorption kinetics is described by linear and nonlinear first-order equations.

Mathematical model. A one-dimensional porous medium consisting of two parts is considered:

1. a zone with mobile pores having porosity m_1, and a stagnant zone saturated with mineralized water having porosity m_2, $m = m_1 + m_2$.
2. the salt distribution within the medium is piecewise homogeneous.

A schematic representation of such a reservoir is shown in Fig. 1.

When a solution with a given concentration is injected into the reservoir, a moving front $x_0(t) = \frac{vt}{m_1}$ is formed — the penetration front of the injected solution through the cross-section $x = 0$. It is assumed that in the stagnant zones the salts are present in dissolved form, and the intradiffusion mass exchange between the mobile and immobile zones occurs infinitely fast [2]. In the absence of a stagnant zone, the reservoir rock may contain salt crystals that instantaneously dissolve when the front $x_0(t)$ reaches them.

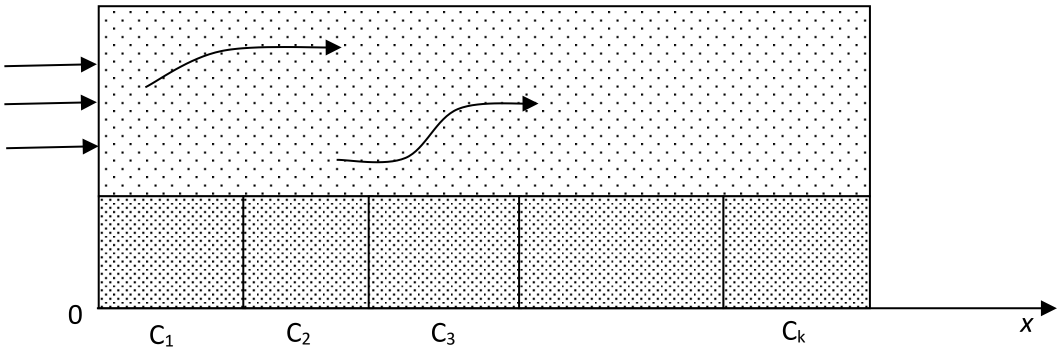

Fig. 1. Schematic diagram of a reservoir whose stagnant zone has piecewise homogeneous salinization.

The solute mass balance equation is written as [3,4]:

$$m_1 \frac{\partial c}{\partial t} + v \frac{\partial c}{\partial x} + \beta \frac{\partial S}{\partial t} = D \frac{\partial^2 c}{\partial x^2}, \qquad (1)$$

where c – is the volumetric solute concentration, m³/m³; D – is the hydrodynamic dispersion coefficient, m²/s; m_1 – is the porosity of the mobile zone; S – is the concentration of adsorbed substance in the mobile zone, m³/kg; v – is the filtration velocity, m/s; x – is the spatial coordinate, m; β – is the bulk density of the porous medium, kg/m³.

The kinetics of nonequilibrium adsorption is described by the first-order equation [5, 6]:

$$\frac{\partial S}{\partial t} = k_1 \frac{m_1}{\beta} c - k_2 S, \quad k_1,\ k_2 \text{ - const.} \qquad (2)$$

As an example, a reservoir whose stagnant zone consists of five sections with boundaries $x_l = 0{,}5l$, $l = \overline{1{,}5}$ is considered. The total reservoir length is $\mathrm{L} = 2{,}5$ m. The transport process is analyzed for time intervals satisfying $x_0(t) \le L$. Each section of the stagnant zone contains an immobile solution with concentration c_i, $i = \overline{1{,}5}$. Pure fluid (without solute) is injected at the inlet $x = 0$. The initial and boundary conditions are:

$$c(0, x) = 0, \qquad (3)$$

$$c(t,0)=0,\ t>0, \qquad (4)$$

$$c(t,x_0(t))=c_0+\sum_{l=1}^{k}(c_l-c_{l-1})\eta(x_0(t)-x_{l-1}),\ \ x_{l-1}\le x_0(t)\le x_l,\ k\le 5, \qquad (5)$$

where c_0 is an arbitrary quantity; $\eta(x)$ is the Heaviside unit step function, $x_0=0$.

Problem (1)–(5) is solved numerically using a finite difference method [7]. The salt concentration distribution in the stagnant zone is specified as piecewise constant:

$c_1=10^{-2}$, $c_2=5\cdot10^{-3}$, $c_3=12\cdot10^{-3}$, $c_4=8\cdot10^{-3}$, $c_5=4\cdot10^{-3}$.

Some numerical results are presented in Fig. 2. The results show that a heterogeneous salt distribution in the stagnant zone leads to a nonuniform distribution of solute concentration in the mobile solution and its adsorption. Concentration discontinuities at the boundaries of stagnant zones are gradually smoothed over time in the profiles of both c and S. The use of nonlinear adsorption kinetics, with all other parameters being equal, results in increased adsorption values, which in turn reduces the solute concentration in the mobile zone.

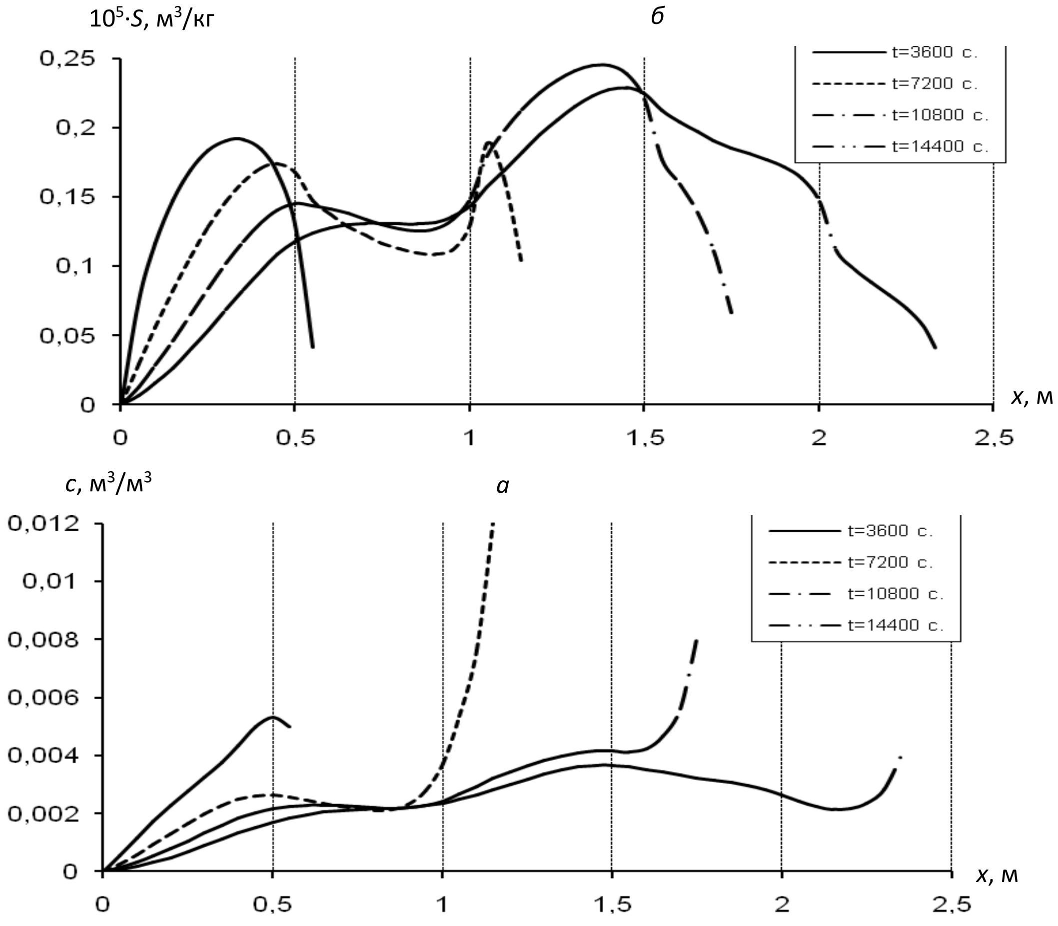

Fig. 2. Concentration profiles $c(a)$, S (b) for $k_1=2\cdot10^{-3}$ c^{-1}, $k_2=5\cdot10^{-4}$ c^{-1}, $\alpha=2000$ c $D=3\cdot10^{-6}$ м^2/c at different moments in time.

Now consider a dry piecewise homogeneous reservoir. In this case, $m_2 = 0$, i.e. there is no stagnant zone with high water retention capacity. The reservoir rock contains crystalline salts with piecewise homogeneous distribution. Upon contact with the injected fluid, the salts instantaneously dissolve and enter the solution. As a result, a diffusive salt flux is formed at the wetting front.

In this case, condition (5) is replaced by:

$$D\frac{\partial c(t,x_0(t))}{\partial x}=\left[c_0+\sum_{l=1}^{k}(c_l-c_{l-1})\eta(x_0(t)-x_{l-1})\right]x_0^{'}(t). \qquad (6)$$

at the wetting front.

Some numerical results for problem (1)–(4), (6) are shown in Fig. 3, corresponding to linear adsorption kinetics.

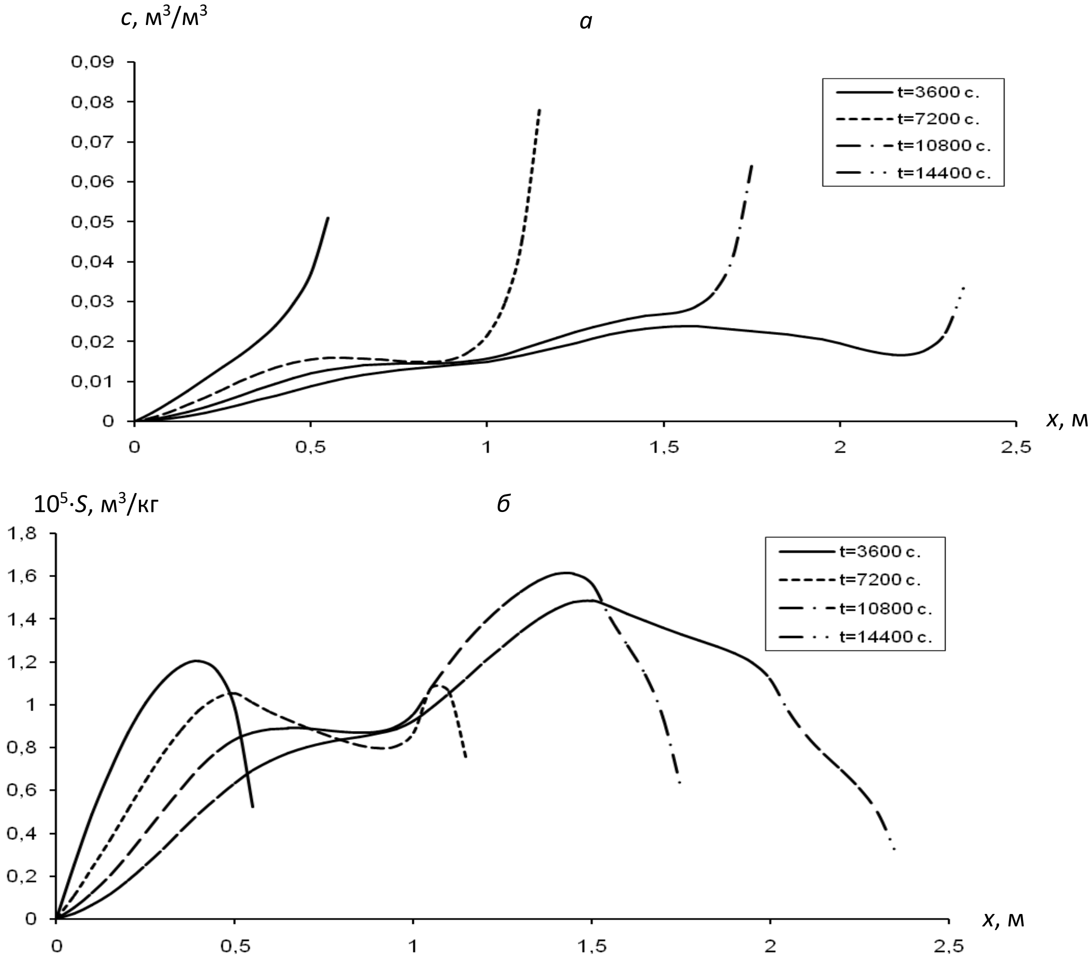

Fig. 3. Concentration profiles $c(a)$, S (b) for при $k_1 = 2\cdot10^{-3}$ c^{-1}, $k_2 = 5\cdot10^{-4}$ c^{-1}, $\alpha = 2000$ c, $D = 3\cdot10^{-6}$ м^2/c at different moments in time.

Conclusion

Specifying the solute flux instead of concentration at the wetting front $x_0(t)$ leads to the formation of a concentration field with significantly higher values. Accordingly, adsorption also increases (Fig. 3).

A decrease in the dispersion coefficient D results in a significant increase in both concentration values and their gradients, which also enhances adsorption. For identical parameters, nonlinear kinetics intensifies adsorption effects, reducing solute concentration compared to linear kinetics. Discontinuities in concentration gradients at the boundaries x_l are rapidly smoothed.

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