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PROPERTIES OF NONLINEAR OSCILLATIONS OF DISSIPATIVE MECHANICAL SYSTEMS WITH A LIMITED NUMBER OF DEGREES OF FREEDOM

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A B S T R A C T

This article studies nonlinear oscillation processes of dissipative mechanical systems with a limited number of degrees of freedom. The properties of dissipative forces, how they affect nonlinear oscillations, the stability of oscillations, the formation of limit cycles, subharmonic and superharmonic resonance phenomena are analyzed. Also, the main mechanisms of nonlinear oscillations are considered using spatial portraits, Lyapunov stability criteria, parametric oscillations, Duffing, Van der Pol and Rayleigh-type models. The energy balance of systems, the effect of the dissipation level on the time evolution of the oscillation amplitude are also covered from a mathematical point of view.

K E Y W O R D S

Nonlinear oscillations, dissipative system, degree of freedom, stability, spatial portrait, resonance, limit cycle, Lyapunov function, Duffing model, Van der Pol oscillations.

Introduction

Oscillation processes of mechanical systems are a fundamental component of many natural and technical systems. Real mechanical systems are often not ideal and have energy losses, i.e. dissipation. In addition, in many processes, forces obey not linear laws, but nonlinear laws. Therefore, the use of nonlinear and dissipative models in the study of oscillations of real systems is important. Systems with a limited number of degrees of freedom constitute a separate section of the theory of oscillations. Such systems include mechanical oscillators, robot manipulator joints, mechanical structures connected by elastic elements, and actuators of automatic control systems. The behavior of these systems is often described not by simple linear models, but by complex nonlinear equations.

The purpose of this article is to analyze dissipative and nonlinear vibrations from a mathematical and physical point of view, to study important mechanisms such as their stability properties, resonance processes, and the formation of limit cycles.

Main part

General description of dissipative systems

A dissipative system is a system in which the energy decays over time, the amplitude of oscillations decreases even in the absence of external influences. Dissipation often arises from the following sources:

1. Internal resistance (internal friction of the material).
2. External friction (air resistance, mechanical friction).
3. Energy consumption of active control systems.

The classical model for a simple one-degree-of-freedom system is:

$$m\ddot{x} + c\dot{x} + kx = 0$$

This is a linear model, and in real systems the following nonlinear additions are often introduced:

$$m\ddot{x} + c\dot{x} + kx + \alpha x^3 = 0$$

This is a Duffing-type nonlinear oscillator.

Causes of nonlinear vibrations.

- Nonlinear vibrations occur in the following cases:
- Nonlinearity of the material
- Large deformations
- Geometric nonlinearity of contact forces
- Various elastic modes of structures
- Aerodynamic effects (flutter, buffeting)
- Backlash in mechanical connections

As a result of these reasons, the vibration equations become more complicated, and their solutions also differ from those in linear systems.

The interaction of dissipation and nonlinearity. The presence of dissipation has a strong effect on the vibration amplitude of the system. In linear systems, energy loss dampens the vibration, while in nonlinear systems:

1. The appearance of a limit cycle
2. Autooscillation (spontaneous oscillation)
3. Complex resonant responses
4. Hysteresis phenomena

Observed.

For example, in the Van der Pol oscillator:

$$\ddot{x}-\mu(1-x^2)\dot{x} + x = 0$$

This equation describes a system with dissipative but nonlinear autooscillations.

Spatial portraits and stability. Spatial portraits are important in the analysis of nonlinear systems. The spatial portrait exhibits the following:

Stable focus
Unstable focus
Limit cycles
Multicycle regimes
Linear convergence
Strange attractors (in some systems)

The Lyapunov function method allows us to evaluate the stability of a dissipative system.
If for some function:

$$\dot{V} < 0$$

then the system is asymptotically stable.

Resonance phenomena. Resonance manifests itself in nonlinear systems as follows:
Superharmonic resonance (in frequency dividers)

Subharmonic resonance (in multipliers)
Combination resonance
The frequency-amplitude characteristic of nonlinear systems is usually curvilinear and has hysteresis. For example, in the Duffing oscillator, the phenomenon of amplitude jumps is observed as a result of the nonlinearity of the stiffness.
This article provides a comprehensive analysis of nonlinear oscillations of dissipative mechanical systems with a limited number of degrees of freedom. The role of dissipation, various sources of nonlinearity, resonance phenomena, spatial portraits, limit cycles, and stability issues are considered. Nonlinear dissipative systems have a much richer dynamic behavior than linear systems, and their analysis is one of the main directions of modern theoretical and applied mechanics. Systems of this type are found in many technical devices, and their correct modeling and control are of great practical importance.

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