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THE DERIVATIVE OF SOME FUNCTIONS BY DEFINITION

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A B S T R A C T	K E Y W O R D S
By taking the derivative of some complex function by definition, it is aimed at students to learn and creatively approach the derivative of a function.	Derivative, Macleron number, function, definition, property, formula.

Introduction

We know $y(x)=\varphi_1(x)^{\varphi_2(x)^{\cdot^{\cdot^{\varphi_n(x)}}}}$ We obtained the derivative of the function of the form from the course of elementary mathematics by logarithmizing its derivative. But we did not find the derivative according to the definition. Let's take the derivative of this function by definition. First of all, let's mention the derivative tariff.

Definition: $f(x)$ function $x\in[a;b]\in R$ is a continuous function on an interval $x_0\in[a;b]$ and $(x_0+\Delta x)\in[a;b]$ function increment $\Delta f(x)$ increment to argument Δx ratio $\Delta x\to 0$ If Theres is limit to the ratio ;

$k=f'(x_0)$ number $f(x)$ funnction x_0 A number is called the point derivative of a function.

We know that $\lim\limits_{x\to 0}\dfrac{f(x)^x-1}{x}=\ln(f(x))$ that $f(x)>0$ is . appropriate.

and have derivatives at that point. The following equality holds,

Macroregulator: $f(x)$ function (a,b) Let a function be defined on an interval, it is $x_0\in(a,b)$ have derivatives at that point $f`, f``, f, \ldots\ldots\ldots f^{(n)}$ The following equality holds,

$$f(x)=f(x_0)+\frac{f`(x_0)}{1!}x+\frac{f``(x_0)}{2!}x^2+\ldots+\frac{f^{(n)}(x_0)}{n!}x^n+r_n(x) \qquad (*)$$

We are given a function $y(x)=\varphi_1(x)^{\varphi_2(x)^{\cdot^{\cdot^{\varphi_n(x)}}}}$, all $\varphi_i(x_0)\neq 0$ va $\varphi_i(x_0)>0$ in this place $i=1,2,3,.....,n$ for a function, we see its first-order derivative:

According to the above definition, we have the following;

$$y_n'(x_0)=\lim_{\Delta x\to 0}\frac{\varphi_1(x_0+\Delta x)^{\varphi_2(x_0+\Delta x)^{\cdot^{\cdot^{\varphi_n(x_0+\Delta x)}}}}+\varphi_1(x_0)^{\varphi_2(x_0)^{\cdot^{\cdot^{\varphi_n(x_0)}}}}}{\Delta x}$$

In this $\varphi_i(x_0)>0$ va $\varphi_i(x_0)\neq 0$ $i=\overline{1,n}$

1-condition: Let's take the derivative in the case where $n=2$ $y_2(x)=\varphi_1(x)^{\varphi_2(x)}$

$$y_2'(x_0)=\lim_{\Delta x\to 0}\frac{\varphi_1(x_0+\Delta x)^{\varphi_2(x_0+\Delta x)}-\varphi_1(x_0)^{\varphi_2(x_0)}}{\Delta x}=$$

$$=\lim_{\Delta x\to 0}\frac{\varphi_1(x_0+\Delta x)^{\varphi_2(x_0+\Delta x)}-\varphi_1(x_0+\Delta x)^{\varphi_2(x_0)}}{\Delta x}+\lim_{\Delta x\to 0}\frac{\varphi_1(x_0+\Delta x)^{\varphi_2(x_0)}-\varphi_1(x_0)^{\varphi_2(x_0)}}{\Delta x}=$$

$$=\lim_{\Delta x\to 0}\varphi_1(x_0+\Delta x)^{\varphi_2(x_0)}\lim_{\Delta x\to 0}\frac{\varphi_1(x_0+\Delta x)^{\varphi_2(x_0+\Delta x)-\varphi_2(x_0)}-1}{\Delta x}+S=$$

According to the above property, we have the following equality

$$=\varphi_1(x_0)^{\varphi_2(x_0)}\varphi_2'(x_0)\ln|\varphi_1(x_0)|+S$$

In this

$$S=\lim_{\Delta x\to 0}\frac{\varphi_1(x_0+\Delta x)^{\varphi_2(x_0)}-\varphi_1(x_0)^{\varphi_2(x_0)}}{\Delta x}=\lim_{\Delta x\to 0}z$$

From the above expression, we get the following

$$\varphi_1(x_0+\Delta x)^{\varphi_2(x_0)}=z\Delta x+\varphi_1(x_0)^{\varphi_2(x_0)} \qquad (1)$$

We expand the left side of equation (1) to Macler's series and lead to equation (1) and get the following expression. $z=\varphi_1'(x_0)\varphi_2(x_0)\varphi_1(x_0+\Delta x)^{\varphi_2(x_0)-1}+O(\Delta x)$

$$S=\varphi_1'(x_0)\varphi_2(x_0)\varphi_1(x_0)^{\varphi_2(x_0)-1}$$

$$y_2'(x_0)=\varphi_1(x_0)^{\varphi_2(x_0)}\varphi_2'(x_0)\ln|\varphi_1(x_0)|+\varphi_1'(x_0)\varphi_2(x_0)\varphi_1(x_0)^{\varphi_2(x_0)-1}$$

$$y_2'(x_0)=\varphi_1(x_0)^{\varphi_2(x_0)}\left[\ln|\varphi_1(x_0)|\varphi_2'(x_0)+\varphi_2(x_0)\frac{\varphi_1'(x_0)}{\varphi_1(x_0)}\right] \qquad (2)$$

2-Condition: Let's take the derivative of the function when $n=3$ $y_3(x)=\varphi_1(x)^{\varphi_2(x)^{\varphi_3(x)}}$

According to the above definition, we have the following;

$$y_3'(x_0)=\lim_{\Delta x\to 0}\frac{\varphi_1(x_0+\Delta x)^{\varphi_2(x_0+\Delta x)^{\varphi_3(x_0+\Delta x)}}-\varphi_1(x_0)^{\varphi_2(x_0)^{\varphi_3(x_0)}}}{\Delta x}=$$

$$=\lim_{\Delta x\to 0}\frac{\varphi_1(x_0+\Delta x)^{\varphi_2(x_0+\Delta x)^{\varphi_3(x_0+\Delta x)}}-\varphi_1(x_0+\Delta x)^{\varphi_2(x_0)^{\varphi_3(x_0)}}}{\Delta x}+$$

$$+\lim_{\Delta x\to 0}\frac{\varphi_1(x_0+\Delta x)^{\varphi_2(x_0)^{\varphi_3(x_0)}}-\varphi_1(x_0)^{\varphi_2(x_0)^{\varphi_3(x_0)}}}{\Delta x}=\lim_{\Delta x\to 0}\varphi_1(x_0+\Delta x)^{\varphi_2(x_0)^{\varphi_3(x_0)}}$$

$$\lim_{\Delta x\to 0}\frac{\varphi_1(x_0+\Delta x)^{\varphi_2(x_0+\Delta x)^{\varphi_3(x_0+\Delta x)}-\varphi_2(x_0)^{\varphi_3(x_0)}}-1}{\varphi_2(x_0+\Delta x)^{\varphi_3(x_0+\Delta x)}-\varphi_2(x_0)^{\varphi_3(x_0)}}y_2(x_0)'+S$$

According to the above property (*) and Case 2, we have:

$$y_3'(x_0)=\varphi_1(x_0)^{\varphi_2(x_0)^{\varphi_3(x_0)}}\ \varphi_2(x_0)^{\varphi_3(x_0)}\left[\ln|\varphi_1(x_0)|\ln|\varphi_2(x_0)|\frac{\varphi_3'(x_0)}{1}+\right.$$

$$\left.+\ln|\varphi_1(x_0)|\varphi_3(x_0)\frac{\varphi_2'(x_0)}{\varphi_2(x_0)}+\frac{\varphi_1'(x_0)}{\varphi_2(x_0)}\right] \qquad (3)$$

Thus, based on equations (2) and (3) above, we write the state of;

$$y_n'(x_0)=\varphi_1(x_0)^{\varphi_2(x_0)^{\cdot^{\cdot^{\varphi_n(x_0)}}}}\ \varphi_2(x_0)^{\varphi_3(x_0)^{\cdot^{\cdot^{\varphi_n(x_0)}}}}\left[\varphi_3(x_0)^{\varphi_4(x_0)^{\cdot^{\cdot^{\varphi_n(x_0)}}}}\left[\varphi_4(x_0)^{\varphi_5(x_0)^{\cdot^{\cdot^{\varphi_n(x_0)}}}}\right.\right.$$

$$\left[\cdots\left[\varphi_{n-2}(x_0)^{\varphi_{n-1}(x_0)^{\varphi_n(x_0)}}\left[\varphi_{n-1}(x_0)^{\varphi_n(x_0)}\left[\ln|\varphi_1(x_0)|\cdots\ln|\varphi_{n-1}(x_0)|\frac{\varphi_n'(x_0)}{1}+\right.\right.\right.\right.$$

$$\left.\varphi_n(x_0)\frac{\varphi_{n-1}'(x_0)}{\varphi_{n-1}(x_0)}\ln|\varphi_1(x_0)|\cdots\ln|\varphi_{n-2}(x_0)|+\frac{\varphi_{n-2}'(x_0)}{\varphi_{n-2}(x_0)}\ln|\varphi_1(x_0)|\cdots\ln|\varphi_{n-3}(x_0)|\right]+$$

$$\left.+\frac{\varphi_{n-3}'(x_0)}{\varphi_{n-3}(x_0)}\ln|\varphi_1(x_0)|\cdots\ln|\varphi_{n-4}(x_0)|\right]+$$

$$+\frac{\varphi_{n-4}'(x_0)}{\varphi_{n-4}(x_0)}\ln|\varphi_1(x_0)|\cdots\ln|\varphi_{n-5}(x_0)|\Big]\cdots\Big]+$$

$$+\frac{\varphi_2'(x_0)}{\varphi_2(x_0)}\ln|\varphi_1(x_0)|\Big]+\frac{\varphi_1'(x_0)}{\varphi_1(x_0)}\Big]$$

The above equality can be proved by the method of mathematical induction.

Example: of the function in the form of $y_3(x)=(x^2+x)^{(2x+1)^x}$ Let's take the first order derivative from the formula given above $x_0=1$
Exactly

$$y_3'(x_0)=\varphi_1(x_0)^{\varphi_2(x_0)^{\varphi_3(x_0)}}\varphi_2(x_0)^{\varphi_3(x_0)}\Bigg[\ln|\varphi_1(x_0)|\ln|\varphi_2(x_0)|\frac{\varphi_3'(x_0)}{1}+$$

$$+\ln|\varphi_1(x_0)|\varphi_3(x_0)\frac{\varphi_2'(x_0)}{\varphi_2(x_0)}+\frac{\varphi_1'(x_0)}{\varphi_2(x_0)}\Bigg]$$

$$y_3'(1)=2^{3^1}\cdot 3^1\cdot\left[\ln(2)\ln(3)+\frac{2}{3}\ln(2)+\frac{3}{2}\right]=24\cdot\left[\ln(2)\ln(3)+\frac{2}{3}\ln(2)+\frac{3}{2}\right]$$

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