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ON THE BEHAVIOUR OF SOME PROBABILISTIC CHARACTERISTICS OF THE OUTPUT OF MULTIDIMENSIONAL RANDOM WALK FROM EXPANDING SETS

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ABSTRACT	**KEYWORDS**
This paper establishes an analog of the well-known theorem of P. J. Bickel and J. A. Yahav on the number of exits of a multidimensional random walk from expanding sets. This theorem and related problems are carried forward for the moment of the first exit.	Multidimensional random walk; expanding sets; moment of the first exit; number of exits.

Introduction

Let $X_1 \ldots X_n$, be independent identically distributed random variables (RV's) with values in $R^d, d \geq 1$. Define $S_0 = 0$ and $S_n = \sum_1^n X_i$ for n≥1. For any Borel set $A \subset R^d$ we set (formally)

$$N(A) = \sum_{n=1}^{\infty} I\,(S_n \in A),\ \ T(A) = \inf\{n, S_n \notin \mathrm{A}\}$$

There are many references dealing with the study of the RV's N(A) and T(A) in the case d=1 (see, for example, the monograph [1]). In the general case, when d > 1, it was proved in (2] that if

$$EN(A) = \sum_{n=1}^{\infty} P(S_n \in A) < \infty$$

for any bounded set A, then

$$Eexp\,\{t\,N(A)\} < \infty$$

for all $|t| \leq t_0$ where t_0, is some positive number. The asymptotic behavior of the moments of N(A) for an expanding set A was investigated in the same paper. As far as the author knows, the distribution of T(A) when d > l has not yet been studied in depth.

Our purpose is to determine the asymptotic behavior of the moments of T(A) on sets of the form $A = A_x = \{y \in R^d, ||y|| < x\}$, where ||·|| is any norm in R^d, as well as the behavior of the “first flight of stairs” $S_{T(A_x)}\ as\ x \to \infty$, In this connection we have proved the following statements.

THEOREM 1. If $E\,||X_1|| < \infty$, then for all k ≥0

$$\lim_{x\to\infty} \frac{ET^k(A_x)}{x^k} = \frac{1}{||EX_1||^k}.$$

This theorem complements a result in [2] on$N(A_x)$.

Let us consider a nondecreasing positive function φ(x) on [0, ∞) that is representable in the form $\varphi(x) = x^l H(x)$ where $l \geq 0$ and H(x) is a slowly varying function in the sense of Karamata.

THEOREM 2. Suppose that $EX_1 \neq 0$ and $E\left||X_1|\right|^2 \varphi\left(||X_1||\right) < \infty$, Then for any ε > 0

$$\int_0^\infty \varphi(x)\ P\{\bar{S}_{T(A_x)} \in A^c_{\varepsilon\, T(A_x)}\}\, dx < \infty$$

Here and in what follows $\bar{S}_n = \sum_1^n (X_i - EX_i)$, and A^c_u is the complement of A_u.

REMARK. By retracing the course of the proof of Theorem 2 it can be shown that

(1) $\int_0^\infty \varphi(x)\, P\{\bar{S}_{T(A_x)} \in A^c_{\varepsilon\, x}\}\, dx < \infty$

It is easy to see that if ε→0, then the left-hand side of (1) converges to ∞, and the asymptotic behavior of the integral with respect to ε is of interest.

Let B be the covariance matrix of the $RV\ X_1$. In this case we have the following assertion, which extends results in [3].

THEOREM 3. Suppose that $EX_1 \neq 0$ and $E||X_1||^{t+2} < \infty$. Then

$$\lim_{\varepsilon \to 0} \varepsilon^{2(1+l)} \int_0^\infty x^1 P\{\bar{S}_{T(A_x)} \in A^c_u\}\, dx = \int_0^\infty x^l P\eta \in A^c_{\sqrt{x}}\}\, dx$$

where η is a normal RV with expectation the zero vector and covariance matrix $||EX_1||^{-1}B$.

We mention some consequences of Theorem 3 when d = 1.

COROLLARY 1. Suppose that $EX_1 \neq 0$ and $EX_1^{l+2} < \infty$ then

$$\lim_{\varepsilon \to 0} \varepsilon^{2(l+2)} \int_0^\infty x^l P\{|\bar{S}_{T(A_x)} > \varepsilon x\} dx = \frac{2\Gamma\left(l + \frac{3}{2}\right)}{\sqrt{\pi}(l+1)} \left(\frac{DX_1}{|EX_1|}\right)^{l+1}$$

COROLLARY 2. If $EX_1 \neq 0$ and $EX_1^2 < \infty$ then

$$\lim_{\varepsilon \to 0} \varepsilon^2 \{\int_0^\infty x^l P\{|\bar{S}_{T(A_x)} > \varepsilon x\} - P\{\bar{S}_{T(A_x)}\}]dx\} = 0$$

The following lemma, which is also of independent interest, can be used to prove the theorems given above.

LEMMA. a) Suppose that $EX_1 \neq 0$. Then for any $\varepsilon > 0$

$$\lim_{x \to \infty} P\left\{\left|\frac{T(A_x)}{x} - ||EX_1||^{-1}\right| > \varepsilon\right\} = 0$$

b) If $EX_1 = 0$, then for sufficiently large C>0

$$\lim_{x \to \infty} P|T(A_x) > Cx = 1$$

REMARK. By using the results in (4] it can be shown that the assertions of the lemma remain in force when the RV X_k takes values in a separable Banach space.

References

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