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BASICS OF DIFFERENTIAL EQUATIONS

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ABSTRACT

Calculus is the mathematics of change, and rates of change are expressed by derivatives. Thus, one of the most common ways to use calculus is to set up an equation containing an unknown function $y=f(x)$ and its derivative, known as a differential equation. Solving such equations often provides information about how quantities change and frequently provides insight into how and why the changes occur. Techniques for solving differential equations can take many different forms, including direct solution, use of graphs, or computer calculations. We introduce the main ideas in this chapter and describe them in a little more detail later in the course. In this section we study what differential equations are, how to verify their solutions, some methods that are used for solving them, and some examples of common and useful equations.

KEYWORDS

Differential equations, solutions, methods, useful equations, specific information.

Introduction

Usually a given differential equation has an infinite number of solutions, so it is natural to ask which one we want to use. To choose one solution, more information is needed. Some specific information that can be useful is an initial value, which is an ordered pair that is used to find a particular solution. differential equation together with one or more initial values is called an initial-value problem. The general rule is that the number of initial values needed for an initial-value problem is equal to the order of the differential equation. For example, if we have the differential equation $y'=2x$, then $y(3)=7$ is an initial value, and when taken together, these equations form an initial-value problem? The differential equation $y''-3y'+2y=4ex$ is second order, so we need two initial values. With initial-value problems of order greater than one, the same value should be used for the independent variable. An example of initial values for this second-order equation would be $y(0)=2$ and $y'(0)=-1$. These two initial values together with the differential equation form an initial-value problem. These problems are so named because often the independent variable in the unknown function is t, which represents time. Thus, a value of $t=0$ represents the beginning of the problem. A differential equation is an equation involving an unknown function $y=f(x)$ and one or more of its derivatives. A solution to a differential equation is a function $y=f(x)$ that satisfies the differential equation when f and its derivatives are substituted into the equation. We already noted that the differential equation $y'=2x$ has at least two solutions: $y=x2$ and $y=x2+4$. The only difference between these two solutions is the last term, which is a constant. What if the last term is a different constant? Will this expression still be a solution to the differential equation? In fact, any function of the form $y=x2+C$, where C represents any constant, is a

solution as well. The reason is that the derivative of x2+C is 2x, regardless of the value of C. It can be shown that any solution of this differential equation must be of the form y=x2+C
This is an example of a general solution to a differential equation. A graph of some of these solutions is given in Figure 1. (Note: in this graph we used even integer values for C ranging between −4 and 4. In fact, there is no restriction on the value of C; it can be an integer or not.)

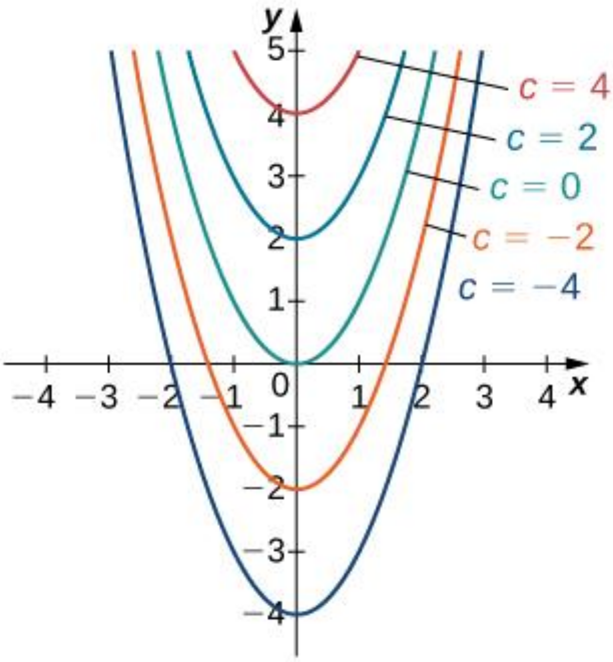

Figure 1. Family of solutions to the differential equation y'=2x

In physics and engineering applications, we often consider the forces acting upon an object, and use this information to understand the resulting motion that may occur. For example, if we start with an object at Earth's surface, the primary force acting upon that object is gravity. Physicists and engineers can use this information, along with Newton's second law of motion (in equation form F=ma, where F represents force, m represents mass, and a represents acceleration), to derive an equation that can be solved. Here are my notes for my differential equations course that I teach here at Lamar University. Despite the fact that these are my "class notes", they should be accessible to anyone wanting to learn how to solve differential equations or needing a refresher on differential equations. I've tried to make these notes as self-contained as possible and so all the information needed to read through them is either from a Calculus or Algebra class or contained in other sections of the notes. An ordinary differential equation (ODE) is an equation that involves some ordinary derivatives (as opposed to partial derivatives) of a function. Often, our goal is to solve an ODE, i.e., determine what function or functions satisfy the equation. A first-order differential equation is defined by an equation: dy/dx =f (x,y) of two variables x and y with its function f(x,y) defined on a region in the xy-plane. It has only the first derivative dy/dx so that the equation is of the first order and no higher-order derivatives exist. The differential equation in first-order can also be written as;

$$y' = f(x,y) \quad \text{or} \quad (d/dx)\, y = f(x,y)$$

The differential equation is generally used to express a relation between the function and its derivatives. In Physics and chemistry, it is used as a technique for determining the functions over its

domain if we know the functions and some of the derivatives. If the function f is a linear expression in y, then the first-order differential equation y’ = f (x, y) is a linear equation. That is, the equation is linear and the function f takes the form f(x,y) = p(x)y + q(x)
where p and q are continuous functions on some interval I. Differential equations that are not linear are called nonlinear equations. This method is similar to the integrating factor method. Finding the general solution of the homogeneous equation is the first necessary step.

$$y' + a(x)y = 0$$

The general solution of the homogeneous equation always contains a constant of integration C. We can replace the constant C with a certain unknown function C(x). When substituting this solution into the non-homogeneous differential equation, we can be able to determine the function C(x). This approach of the algorithm is called the method of variation of a constant. However, both methods lead to the same solution.

Now multiplying both sides by;

$$\mu(x) = e^{\int P dx} = e^{\int 2x dx} = e^{x^2}$$

$$e^{x^2} + 2xe^{x^2}y = xe^{x^2}$$

$$\frac{d}{dx}(e^{x^2}y) = xe^{x^2}$$

Now integrating both the sides, we get;

$$e^{x^2}y = \int xe^{x^2}dx$$

$$e^{x^2}y = \frac{1}{2}e^{x^2} + c$$

$$y = \tfrac{1}{2} + ce^{-x^2}$$

In calculus, a differential equation is an equation that involves the derivative (derivatives) of the dependent variable with respect to the independent variable (variables). The derivative represents nothing but a rate of change, and the differential equation helps us present a relationship between the changing quantity with respect to the change in another quantity. y=f(x) be a function where y is a dependent variable, f is an unknown function, x is an independent variable. Here are a few differential equations.

(dy/dx) = sin x
(d2y/dx2) + k2y = 0
(d2y/dt2) + (d2x/dt2) = x
(d3y/dx3) + x(dy/dx) - 4xy = 0
(rdr/dθ) + cosθ = 5

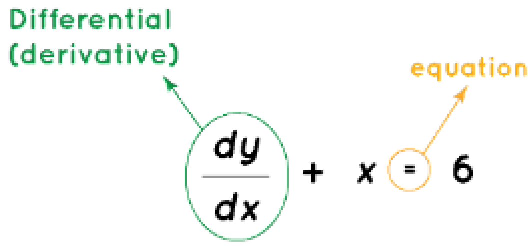

Figure 2. Differential equation.

If a differential equation is expressible in a polynomial form, then the integral power of the highest order derivative that appears is called the degree of the differential equation. The degree of the differential equation is the power of the highest ordered derivative present in the equation. To find the degree of the differential equation, we need to have a positive integer as the index of each derivative. A differential equation in which the degree of all the terms is the same is known as a homogenous differential equation. In general they can be represented as P(x,y)dx + Q(x,y)dy = 0, where P(x,y) and Q(x,y) are homogeneous functions of the same degree. The differential equation has infinitely many solutions. Solving a differential equation is referred to as integrating a differential equation since the process of finding the solution to a differential equation involves integration. A solution of a differential equation is an expression for the dependent variable in terms of the independent variable which satisfies the differential equation. The solution which contains as many arbitrary constants is called the general solution. If we give particular values to the arbitrary constants in the general solution of the differential equation, the resulting solution is called a Particular Solution. The result of eliminating one arbitrary constant yields a first-order differential equation and that of eliminating two arbitrary constants leads to a second-order differential equation and so on. Let us understand solving the differential equation by an example. Ordinary differential equations applications in real life are used to calculate the movement or flow of electricity, motion of an object to and fro like a pendulum, to explain thermodynamics concepts. Also, in medical terms, they are used to check the growth of diseases in graphical representation. Differential equations are useful in describing mathematical models involving population growth or radioactive decay.

Conclusion

An equation that contains the derivative of an unknown function is called a differential equation. The rate of change of a function at a point is defined by the derivatives of the function. A differential equation relates these derivatives with the other functions. Differential equations are mainly used in the fields of biology, physics, engineering, and many. The main purpose of the differential equation is for studying the solutions that satisfy the equations and the properties of the solutions. Let us discuss the definition, types, methods to solve the differential equation, order, and degree of the differential equation, types of differential equations, with real-world examples, and practice problems.

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