



ASYMPTOTIC ESTIMATES ON A SMALL PARAMETER IN HARTMANN-WINTNER LAW OF ITERATED LOGARITHM

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ABSTRACT	KEYWORDS
An asymptotic estimate of the convergence velocity in the law of repeated logarithm in the form of convergence of series from the probabilities of large evasions is obtained.	the law of repeated logarithm, the convergence of the series, the number of outputs, the normal law, the speed of convergence in the central limit theorem

Introduction

Let $\{X, X_n; n \geq 1\}$ be independent equally distributed random variables (s.v.) with zero mean and finite variance

Put

$$S_0 = 0, \quad S_n = \sum_{k=1}^n X_k, \quad n = 1, 2, \dots$$

The classical Hartmann–Wintner repeated logarithm law is known:

$$\lim_{n \rightarrow \infty} \pm \frac{S_n}{\sqrt{2n \ln \ln n}} = 1$$

The task of studying the rate of convergence in the above ratio is of great interest, and at this time a large number of articles have been published on this subject, the results of which have various applications in interpretation.

Analysis of the results shows that research on this task was conducted in two directions:

A. For a given function, estimating probabilities of a form: $H(x)$

$$P_n = P\{S_k \geq H(k) \text{ для некоторого } k \geq n\} \text{ при } n \rightarrow \infty$$

In the works of D. Darling and G. Robbins [1], G. Robbins and D. Sigmund [2] in the assumption established, $E \exp hX < \infty$ для $|h| < h_0$

A what

(a) There is a sequence

$H(n) \sim \sqrt{2n \ln \ln n}$ such that with $n \rightarrow \infty$

$$P_n = O\left(\frac{1}{\ln \ln n}\right);$$

c) if, for some $a > 1$, $H(n) \sim \sqrt{2an \ln \ln n}$
then at $n \rightarrow \infty$

$$P_n = O\left(\frac{1}{\ln^{a-1} n}\right)$$

The most accurate results in terms of descending order were established by W. Strassen [3].

He proved that if the function $H(x)$ is such that

$$E \exp hX < \infty \text{ для } |h| < h_0 \ (h_0 > 0)$$

$$x^{-1/2} H(x) \uparrow, H(x) \leq x^c, c < 3/5$$

$$\frac{H'(x)}{H(y)} \rightarrow 1 \text{ при } \frac{x}{y} \rightarrow 1, \text{ то при } n \rightarrow \infty$$

$$P_n \approx \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{H'(t)}{\sqrt{t}} \exp\left\{-\frac{H^2(t)}{2t}\right\} dt$$

Strassen's research continued in the works of T.L. Lai V.S. Chow [4], T.L. Lai and K. Lan [5], M.U. Gafurov and A.D. Slastnikov [6], M.U. Gafurov [7]. In particular, it is noted that it follows from the overall results [6] that if , then $H(x) \approx \sqrt{(2 + \varepsilon) \ln \ln n}$

$$P_n \leq C \sqrt{\ln \ln n} \ln^{-\frac{\varepsilon}{2}} n, \quad C > 0, \forall \varepsilon > 0$$

which is the best, in the sense of the order of descending to zero probability P_n .

B. Associate the non-negative, monotonously increasing function and non-negative function with the series $H(x)f(x)$

$$R = \sum_{n=1}^{\infty} \frac{f(n)}{n} P(|S_n| > H(n)).$$

It should be noted that with proper choice of function and convergence of the series characterizes the rate of convergence in the law of repeated logarithm. For special cases, function and convergence were investigated by J. S. Miller. Davis [8].

Davis proved that if the conditions are met, the following statements are equivalent: $EX = 0, EX^2 \ln |X| \ln \ln |X| < \infty$

$$\sum_{n=1}^{\infty} \frac{\varphi^2(n)}{n} P\{|S_n| > \sqrt{n}\varphi(x)\} < \infty$$

and

$$\int_1^\infty \frac{\varphi^2(x)}{x} e^{-\frac{\varphi^2(x)}{2}} dx < \infty$$

where $\varphi(x) > 0, \varphi(x) \uparrow, x \in [1, \infty)$.

This result was summarized and reinforced in the work of M.U. Gafurov [14] for a broad class of functions and distribution functions.

Let be strictly increasing functions defined on satisfying conditions

$$\frac{f(x)}{\varphi^2(x)} \uparrow, \quad \frac{f(x)}{\varphi^3(x)} \downarrow, \quad x > x_0, \quad x_0 > 0 \quad (\cdot)$$

Let's put and denote the function inverse of $H(x) = \sqrt{x}\varphi(x)H^{-1}(x)H(x)$

Theorem 1 [14]. Let the conditions be met,

$$EX = 0, EX^2 = 1 \text{ и } E[H^{-1}(|X|)f[H^{-1}(|X|)\ln|X|]] < \infty \quad (\cdot)$$

Then

$$R < \infty \Leftrightarrow \int_1^\infty \frac{f(x)}{\sqrt{x}H(x)} \exp\left\{-\frac{H^2(x)}{2x}\right\} dx < \infty$$

In the same work, a special case is highlighted when

$$\varphi(x) = \sqrt{(2 + \varepsilon) \ln \ln x}, \quad f(x)$$

Then, it is not difficult to understand that, the condition takes the form

$$H^{-1}(x) = \frac{x^2}{\sqrt{(2 + \varepsilon) \ln \ln x}} E[H^{-1}(|X|)f[H^{-1}(|X|)\ln|X|]] < \infty \quad EX^2 \ln|X| < \infty$$

Let's introduce the functionality into consideration

$$\lambda_\varepsilon = \sum_{n=3}^\infty \frac{\ln \ln n}{n} I(|S_n| > \sqrt{(2 + \varepsilon)n \ln \ln n})$$

where is the event indicator, which is the "weighted number" of outputs of the random wandering trajectory beyond the two-way boundary $\pm\sqrt{(2 + \varepsilon)n \ln \ln n}$. It is easy to understand that under p.n. the naturally occurring problem of asymptotic behavior

in dependence was solved for the first time in [14]: $\lambda_\varepsilon \uparrow \infty$ when $\varepsilon \downarrow 0$

Theorem 2 [14]. Let $EX = 0, EX^2 = 1, EX^2 \ln|X| < \infty$.

Then

$$\lim_{\varepsilon \downarrow 0} \varepsilon^{3/2} E\lambda_\varepsilon = \sqrt{2}$$

The statements of these theorems 1,2 and the methods of their proof attracted the attention of specialists in Germany, China, Sweden and others, and they stimulated the emergence of a number of new refinements and generalizations in this direction [14] ... [15]. Regarding the approval of theorem 2, we also add that back in 1981 at a seminar on probability theory of the Steklov Mathematical Institute of the Russian Academy of Sciences, Academician Yu.V. Prokhorov expressed the task of finding further members of the asymptotic representation. However, this task has not yet been solved. It is to the solution of the task that the present work is devoted.

Theorem 3. Under the conditions of theorem

$$\varepsilon\sqrt{\varepsilon}E\lambda_{\varepsilon} = \sqrt{2} - \frac{3\sqrt{2}}{4}\varepsilon + o(\varepsilon)$$

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